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ABSTRACT

This book contains the tenth chapter of a pilot mathematics sequence for the seventh and eighth grades. The content of the sequence is to serve as a vehicle for the development of relevant computational skills, mathematical reasoning, and geometric perception in three dimensions and is to reflect the application of mathematics to the social and natural sciences. The material is divided into five types of sections: (1) activities; (2) short reading sections; (3) questions; (4) sections for the student with a weaker background; and (5) sections for the strongly motivated student. The material in chapter ten includes probable and improbable events. (MN)

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MATHEMATICS

FOR JUNIOR HIGH SCHOOL

pilot edition — *chapter 10*

Boston University Mathematics Project

SE 021 281

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10. PROBABLE AND IMPROBABLE EVENTS

SECTION 1 PIE CHARTS

An opinion poll was taken to find out if the President did his job well. At one time 65 percent said "yes" and 35 percent said "no." A year later another poll was taken. This time the answers were 80 percent "yes" and 20 percent "no."

The two pie charts in Figure 1 have the same information as the sentences in the above paragraph. Yet most people say the pie chart says it in a more convincing way.

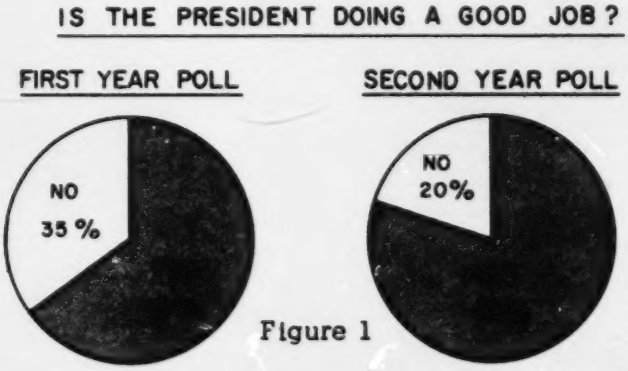


Figure 1

How are pie charts made? How are the angles corresponding to the percentages calculated? The whole circle, consisting of 360°, corresponds to 100%. The angle corresponding to any given percent-

age is that same percentage of 360°. For example, the angle corresponding to 30% is 30% <u>of</u> 360°. As you have seen before, the word "of" calls for multiplication. Thus

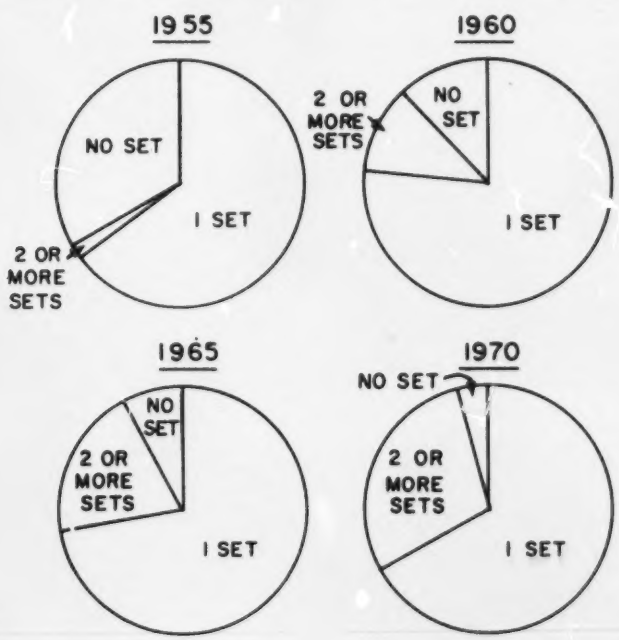
angle in pie chart = percentage × 360°

1. Draw a pie chart that has angles corresponding to 25%, 15%, and 60%.

2. The pie charts in Figure 2 indicate the percentages of households in the United States with 0, 1, or 2 or more television sets. No numbers for the percentages are given.

Figure 2

PERCENTAGES OF HOUSEHOLDS WITH TELEVISION SETS



(a) Estimate the percentages of households with 1 television set during each of the four years shown.

(b) Estimate the percentages of households having two or more sets during each of the four years.

(c) Do you think that the total number of television sets decreased from 1960 to 1965?

3. (a) How can you calculate the percentages on a pie chart after measuring the angles with a protractor?

(b) Check your estimated answers to Question 2 by actual measurements and calculation.

How accurately can you measure an angle on your protractor? If, for example, you can only read angles to within 1°, there is no reason to calculate angles to a higher accuracy. Decide how accurately you can measure angles with your protractor. Then carry out the calculations in the following questions to only that accuracy.

4. The following table lists the percentages of families of a given size in the United States for the year 1972.

Number of Persons	Percentage
2	35.4
3	21.2
4	19.7
5	11.9
6	6.2
7 or more	5.5
	total 99.9

(a) Calculate the corresponding angles and draw the pie chart.

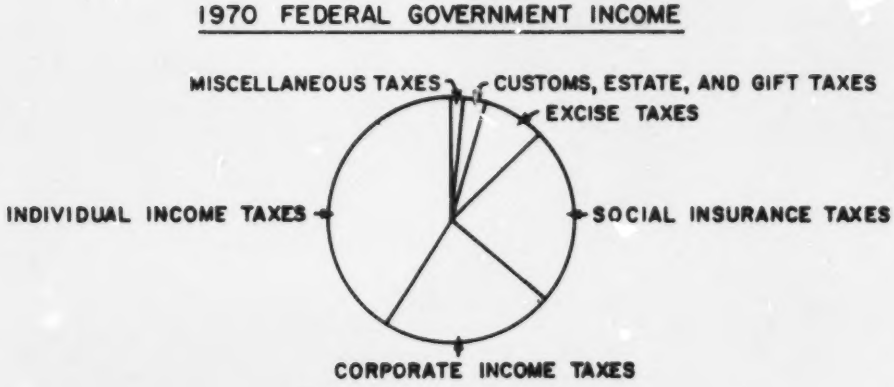
(b) How do you explain the fact that the percentages do not add up to 100%?

(c) Did this fact affect your pie chart?

5. The percentages in the following table stand for the federal government's income for 1970. From this data an artist constructed the pie chart in Figure 3. Find the error he made in constructing this pie chart.

Individual income taxes	46.7%
Corporation income taxes	16.9%
Social insurance taxes	23.4%
Excise taxes	8.1%
Customs, estate, and gift taxes	3.1%
Miscellaneous taxes	1.8%

Figure 3



6. The table below lists the number of cars produced in the United States by each company during 1972. Make a pie chart showing the percentage of cars produced by each company.

American Motors Corp.	279,132
Chrysler Corp.	1,367,354
Ford Motor Co.	2,400,871
General Motors Corp.	4,775,344
Checker Motors Corp.	5,504
	8,828,205

SECTION 2 SODA TASTING

Of course you can tell the difference between a cola, an orange soda, and a lemon soda! But which of your senses enables you to tell the difference? Is it by seeing the different colors? Is it by smelling the different odors? Or is it by tasting the different flavors?

Whenever you are faced with many questions it is best to answer them one by one. In this experiment you will be given a sample of one of these sodas and you will be asked to tell what flavor it is. However, you must close your eyes and hold your nose until after you have made your guess. In this way you will get an idea of how important taste is alone.

A student volunteer will record what flavors each of you will be given and what guess each of you will make. A convenient way to record this information is to write the name of the student, the initials of the soda he was given, and what he guessed. For example, John Lamb was given cola and guessed "orange." This would be recorded as

John Lamb (C, O)

It is important to keep the correct order. Reversing it would change the meaning. The class results will be distributed after the experiment is completed.

7. Make a tally of all the times when students were given cola. How many said it was cola? Orange? Lemon soda?

8. From the tally of guesses when cola was given, what percentage of students guessed cola? What percentage guessed orange? What percentage guessed lemon?

9. Make a pie chart of your answers for Question 8. What percentage guessed correctly? Incorrectly?

10. What would you expect the pie chart to be like if the whole class could recognize cola by taste alone?

11. Assume that nobody in the class could recognize the cola by taste alone. They just said "cola," or "orange," or "lemon" -- whatever came into their heads first. What do you think the pie chart would look like?

SECTION 3 EQUALLY PROBABLE EVENTS

If you toss a coin, is it more likely to come out heads or tails? If the coin is bent, it may be more likely to land on one side than on the other. But for a well-made coin we see no reason why one side is more likely to be up than the other. Whenever we see no reason why one event should be more or less likely than another, we say that the two events are <u>equally likely</u> or <u>equally probable</u>. (An "event" here is anything that happens.)

We think of heads or tails as equally likely events when we toss one coin. What is likely to be the result of, say, 1000 tosses? We cannot predict exactly what will happen because each toss can go either heads or tails. But we can feel pretty sure that the result will be close to half being heads and half being tails.

Now let us turn our thoughts to actions that can have three different results. Imagine a bag containing red, blue, and green marbles. If you pick one marble it may be red, blue, or green. Suppose there is an equal number of marbles of each color in the bag. Also assume that all the marbles are the same size and weight and are well mixed. Therefore we have no reason to believe that it is more likely to pick a red marble than a green or a blue one. Again we say that it is equally probable to pick a red, a blue, or a green marble.

If we were to pick a marble repeatedly, we could expect about a third of the total to be of each color.

12. (a) Suppose you throw a perfect die. Are you as likely to throw a one as a six? Why or why not?

(b) You throw the die many times. In what fraction of the throws is each number likely to appear.

(c) Make a pie chart to illustrate your answer to part (b).

13. (a) With a perfect die, is it as probable to throw an even number as it is an odd number?

(b) If you throw the die many times what fraction of the throws will turn out even?

(c) Draw a pie chart to illustrate your answer to part (b).

14. Suppose that a friend tells you that she can predict whether your next throw of a die will produce an even or an odd number. You challenge her by asking her to predict the result of 100 throws. She turns out to be right in about half of them. Do you now believe her?

15. A true-false test in French has an equal number of true and false statements. A student who does not know any French

takes the test. What fraction of the answers is he likely to get right?

16. Suppose that with closed eyes and nose it is equally probable to think of the cola as cola, orange, or lemon. Draw a pie chart of the results.

17. Compare your pie chart for Question 16 with the pie chart for Question 9. Can you prove that the class can recognize cola by taste alone?

18. Tally all the times when students were given orange soda. How many said it was orange? Cola? Lemon? What are the percentages? Was your class any better in recognizing orange soda?

19. Repeat the entire analysis of the class data for the case of lemon soda. What do you conclude?

SECTION 4 COUNTING EQUALLY PROBABLE EVENTS

You need not rely on feeling to guess that out of a large number of coin tosses about half will be heads and half tails. We can provide a reason for it.

Let us start with a single toss. In this case heads (H) or tails (T) is equally probable. Now we look at two tosses. There are four possible results: two heads, a head followed by a tail, a tail followed by a head, and two tails. In Table 1 these results are conveniently written in shorthand. The left-hand letter of each pair stands for the result of the first toss.

TABLE 1

(H H)	(H T), (T H)	(T T)
1	2	1

For each toss H and T are equally probable. Furthermore, the result of the second toss does not depend on the result of the first toss. Therefore, each pair of results in Table 1 stands for equally probable events.

There is only one way to get two heads, but there are two ways to get one head and one tail. This is indicated by the numbers in Table 1. Therefore, it must be twice as likely to get one head and one tail as it is to get two heads.

Let us go one step further and count the number of equally probable results of three tosses. This is done in Table 2.

TABLE 2

(H H H)	(H H T), (H T H), (T H H)	(H T T), (T H T), (T T H)	(T T T)
1	3	3	1

Again, the letter on the left stands for the result of the first toss. Note that there are three ways to get two heads and one tail, but only one way to get three heads. Thus it is three times as likely to toss two heads and one tail as three heads.

We could go on counting equally probable results for larger and larger numbers of tosses, but this is not necessary. Our re-

sults so far suggest a pattern that allows us to find the answer for larger numbers of tosses. Table 3 is a summary of our results so far and the extension to larger numbers.

TABLE 3

Total Number of Tosses	Number of Heads						
	0	1	2	3	4	5	6
1	1	1					
2	1	2	1				
3	1	3	3	1			
4	1	4	6	4	1		
5	1	5	10	10	5	1	
6	1	6	15	20	15	6	1

"Zero heads" means all tosses turned out tails. Similarly, "one head" means that all the other tosses came out tails, and so on. Table 3 shows a clear trend: As the total number of tosses increases, it becomes much more likely to have about half the tosses being heads than about all being heads.

20. (a) Figure out what should be the next line in Table 3.

(b) How much more likely is it to get 4 heads out of 7 tosses than it is to get 7 heads?

(c) How much more likely is it to get 4 heads out of 7 tosses than it is to get 6 heads?

21. When you throw a perfect die, each number is equally probable to be on top. Suppose you throw a pair of perfect dice.

(a) What are the equally probable combinations of the results?

(b) How many combinations are there whose sum equals 8? How many combinations are there whose sum is 12?

(c) How many times as likely is it to throw an 8 as a 12?

SECTION 5 AVERAGES

The heights of 20 students to the nearest centimeter were measured and the results were

130	167	142	145	161
152	137	143	165	163
135	140	170	157	164
167	185	143	160	164

How can we summarize this information? Is there a single number that can give us a general idea about the heights of all 20 students? One such number is the <u>average</u>. To calculate the average we add all the heights together and divide by the number of students.

$$\text{Average height} = \frac{\text{sum of all heights}}{\text{the number of students}}$$

We find from our calculation that the average height is 154 cm.

Whenever we summarize a set of measures with a single number, such as an average, we lose some information. Knowing that the average height of 20 students is 154 cm does not give us as much information as knowing the height of each student. For example,

knowing only the average we could not tell how tall the shortest student in the class is. But in many cases the average provides enough information and is easier to remember.

22. Peter rolls 5, 1, 2, 5, 3, 4, and 6 with a die. What is his average score?

23. Use the following table to answer the following questions.

Farm Statistics

Year	Number of Farms (in thousands)	Farmland (in millions of acres)
1930	6,546	987
1950	5,648	1,202
1970	2,954	1,103

(a) Find the average number of acres per farm for 1930, 1950, and 1970 separately.

(b) From the data given, describe what has been happening to the size of farms.

24. A bowling league of 8 teams meets once a week. Each team has 6 players and each player bowls 3 games.

(a) The top bowler on Team A had scores of 210, 230, and 181 during one week. What was his average score?

(b) Another player on Team A bowled a 173 and a 161 on the first two games. If his average for the day was 155, what must have been his score on the third game?

(c) During that same week the other 4 players on Team A had averages of 135, 160, 143, and 124. What was the average for the whole team?

25. In 10 rolls of a die, Joe averaged 4 points. Using the same rules, Buddy averaged 4.5 points in 20 rolls. Taken together, what was the average number of points for both of them?

26. If Jim got a 96 and an 80 on two tests, and if 90 is an "A," what must he get on the next test to bring his average up to an "A"?

27. Erika wanted to figure out her test average for the year. She had kept a record of her averages for each marking period. Use the following table to find her test average for the year.

Marking Period	Number of Tests Taken	Average
1	2	78
2	3	80
3	5	75
4	2	90

SECTION 6 WHICH GAME RULE IS BEST?

Here is a game for you to play with a friend. You each roll a die and use one of the scoring rules in Table 4 to find out how many points you get. For example, if you are using rule D and you roll a 3, you get 5 points. On the other hand, if you choose rule A and you roll a 3, you get 3 points.

Take a few moments to study the rules. Which do you think is best for you?

You must choose which rule you want to use before you start the game. Both of you may choose the same rule if you wish. The object is to score as many points as possible.

TABLE 4

Showing on Die	Points A	B	C	D
	1	4	0	8
	2	4	0	5
	3	4	0	5
	4	4	18	4
	5	4	0	4
	6	4	0	0

The higher total score at the end of 20 rolls of the die wins the game. Your teacher will record each person's score and the game rule used.

Which do you think is the best rule now that you have played the game? How sure are you?

Look at the class data. Were all the rules used? For the rules that were used, which rule do you think is best? What is the average score of all the students playing rule A? What is the average score of all the students playing rule B? Rule C? Rule D? Use these averages to decide which rule is best.

SECTION 7 BAR GRAPHS

A student took a survey of 138 students in his school to find out what the most popular soft drink is. He put the results of the survey in a table.

	Orange	Root Beer	Cola	Ginger Ale	
Number of Students	47	24	46	21	Total 138

Another way to display this information is on a bar graph, as in Figure 4.

Note that the bar graph has a title so that we know what information is shown. Also note that the label "Number of Students" is used to show what the numbers stand for.

A bar graph can also be drawn sideways if you prefer, as shown in Figure 5.

Figure 4

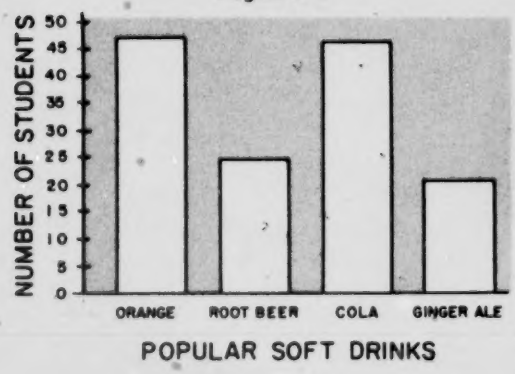
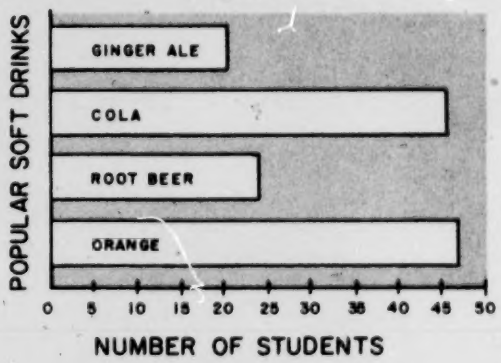


Figure 5



Either way the bar graph displays the same data as the table but in a way that makes the information easier to see at a glance.

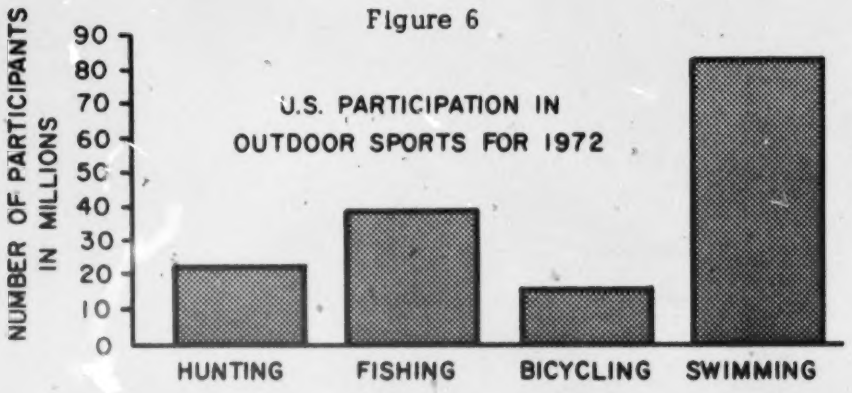
28. The data in the bar graph in Figure 6 are projections based on a survey of 4000 Americans, twelve years old and older. Use these data to answer the following questions.

(a) Approximately how many Americans fished in 1972?

(b) Of the four sports listed, which is the most popular?

(c) About 10 percent of the total population goes bicycling. About what percent goes swimming?

Figure 6



29. From the table below make a bar graph showing the number of people who speak each of the five most commonly spoken languages.

Language	Number of Speakers
Chinese	750 million
English	300 million
Russian	200 million
Spanish	200 million
Hindi	180 million

30. Gwendolyn took a survey of 150 students in her school. She asked, "On the average, how much time do you spend on homework each night?" Her tabulated results are:

Number of Minutes	0	5	10	15	20	25	30	40	45	60	75	90
Number of Students	6	10	13	28	21	13	18	5	15	12	6	3

(a) Make a bar graph of the results.

(b) What amount of time was most frequently given?

(c) What percent of the students spend more than 30 minutes on homework? Less than 15 minutes? 15 minutes through 30 minutes?

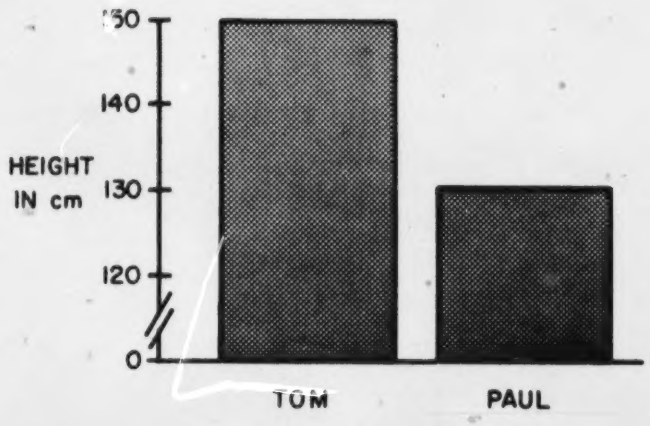
31. Tom and Paul were comparing their heights. Tom insisted that he was much taller than Paul. To prove his point he graphed their heights as follows:

(a) Is Tom actually taller than Paul? How many centimeters taller?

(b) The bar representing Tom's height is how many times as large as the bar representing Paul's height?

(c) Tom's height is how many times as large as Paul's height?

(d) Make another bar graph that more accurately represents the comparison of their heights.



SECTION 8 COMPARING GAME STRATEGIES

In all likelihood some rules for the game in Section 6 were chosen by more students than were other rules. For example, very few students may have chosen rule B. You already know from your work with codes that small samples are not reliable. To be able to judge each rule you need larger samples. To get them, play each rule once. Then collect the results from the whole class.

Work with one of your classmates. Roll a die ten times and record your results in a table like Table 5.

TABLE 5

Roll	Results of Roll	Score			
		A	B	C	D
1	4	4	4	18	4
2	1	1	4	0	8
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
10	3	3	4	0	5

After you have rolled the die ten times, find the total score for each rule. Your teacher will make bar graphs to display the scores of the whole class.

What is the highest and lowest score for each rule? What is the average score for each rule? If you were to win a penny for each point you scored, which rule would you rather play? Why?

Look at the bar graphs. Which rule has the greatest range of scores? Which has the least? For which rule do you think luck is most important? Why?

Is there a rule that always gives a better score than other rules? Is it possible to win by using any of the rules?

SECTION 9 EXPECTATIONS

Is there some way that you could decide which is the best rule without playing any games? One way to do this is to figure out what score you can expect on each game if you were to play the games many times.

Suppose you are using rule C (Table 5). You roll the die many times and write down your score for each roll. If the die is perfect, the numbers 1 through 6 are equally likely to come up. On what fraction of the rolls would you therefore expect to get a score of 18?

You receive a score of 18 when a 4 comes up. A 4 is likely to appear $\frac{1}{6}$ of the time. Therefore, you would expect to get a score of 18 on about $\frac{1}{6}$ of the throws. On the other hand, you would expect a score of 0 on about $\frac{5}{6}$ of the throws. Why?

On a large number of rolls, say 6000, you would expect to get a score of 18 about 1000 times and a score of 0 about 5000 times. For 6000 throws, using rule C, you would probably, therefore, have a total score of about 18,000 points. This is an average of 3 points

for each roll. By calculating the expected number of points for each roll for the other rules, we can compare the rules and decide which is the best.

32. (a) Suppose you are using rule A. In what fraction of throws would you expect a score of 1? A score of 2?

(b) For 6000 rolls how many times would you expect a score of 1? A score of 2?

(c) For 6000 rolls what would probably be your total score for rule A?

(d) What is the average number of points per roll for rule A?

33. What is the average number of points per roll for rule B?

34. (a) For rule D, in what fraction of rolls would you expect a score of 8? A score of 5? A score of 4? A score of 0?

(b) For 6000 rolls how many times would you expect to get each of the scores of part (a)?

(c) For 6000 throws what would probably be your total score for rule D?

(d) What is the average number of points for each roll for rule D?

(e) Which rule has the highest expected average for each roll? How does this agree with what you found in Section 5?

35. Which rule has the highest expected average for each roll? How does this agree with what you found in Section 5?

SECTION 10 ESTIMATING FRACTIONS OF A LINE SEGMENT

How well can people mark the midpoint of a line segment? Are they as likely to be off to the left as they are to the right? Is

it just as easy to mark the one-third point on a segment? Here is an experiment that will give some answers.

You have sheets of paper with the segments already drawn. On one sheet mark the point that you think divides the segment in half. On the other segment mark what you think is the one-third point from the left without looking at the first sheet.

Exchange papers with a classmate. Using a centimeter ruler, mark the actual one-half or one-third points on the appropriate sheets. For each segment measure to the nearest tenth of a centimeter the distance between the estimated and the actual points. This will tell you the size of the error in the estimate. (If your mark fell within 0.1 centimeter of the actual point, you can consider your mark to be correct.) If you made an error be sure to write down whether the mark was to the right or to the left of the actual point.

(a) How many students in your class marked the one-half point correctly? How many missed to the left? How many missed to the right?

(b) How many students in your class marked the one-third point correctly? How many missed to the left? How many missed to the right?

(c) Of all the students who did not mark the one-half point correctly, what percentage was off to the left? To the right? Show the class results on a pie chart.

(d) Suppose people were as likely to be off to the left as to the right. What would a pie chart look like under this assumption? Is this assumption valid for your class?

(e) Of all the students who did not mark the one-third point

correctly, what percentage was off to the left? To the right? Again show the class results on a pie chart. Are people as equally likely to be off to the left as to the right when making the one-third point?

(f) How well did you do? Did you guess the one-half and one-third points as accurately as your classmates? Make bar graphs to display the errors made by the entire class. For the bar graphs consider only the size of error and disregard whether it was to the left or to the right.

(g) How many students in your class had an error greater than yours? What percentage of the class is this?

(h) Find the percentage of the class that had an error less than or equal to yours. This is called the <u>percentile rank</u>. What is your percentile rank?